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THE LONGWOOD GARDENS ANALEMMATIC SUNDIAL

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The analemmatic sundial at Longwood Gardens was originally built in 1939, based on the design of a sundial at the Cathedral of Brou, France, which was originally built in the early 16th century. The Longwood sundial is 24 by 37 feet with a gnomon movable along an analemma. The hour markers are moveable for daylight saving time. However, the sundial did not tell time correctly. The US Naval Observatory was contacted by the director of maintenance at Longwood Gardens about the problem. After measuring the sundial and computing a solar ephemeris for the location, computations were made to determine the correct locations for the gnomon. Analemmas very different from the current ones were determined for the morning and afternoon hours. After contact with a historian in Brou, France, it was determined that the sundial there had been rebuilt twice, and the current design did not tell the correct time either. That inaccurate sundial design from France was revised by computer technology to give an analemmatic sundial that told mean solar time directly, including the correction for the equation of time.

INTRODUCTION

Horizontal sundials are perhaps the most common type of dial. Traditionally, these dials have a flat horizontal face and a fixed gnomon or *style* (the index casting the shadow) directed along the local meridian. The style is often triangular, with the upper edge angled according to the latitude of the dial. This type of dial tells time by comparing the direction of the style's shadow with directional rulings along the horizontal dial face.

The classical *analemmatic sundial* is a less-common type of dial, usually horizontal, with a *vertical* rod or pole as the gnomon which must be repositioned along the dial's meridian line according to the time of year in order for the dial to indicate the intended time. This kind of dial is read from the intersection of the gnomon's shadow with points on a graduated ellipse marking off the time of day. The analemmatic sundial gets its name from the fact that the word *analemma* originally referred to a type of orthographic projection technique which was useful for dividing the elliptical dial into equal solar hours.[1]

Most sundials, including analemmatic dials, intrinsically indicate local solar time. To realize local clock time from them, corrections must be applied for daylight-saving (summer) time, the difference between local time and standard time, and the difference between mean and apparent solar time over the course of the year. This last correction is commonly known today as the *equa-*

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tion of time, which ironically, when plotted against solar declination is also known as an *analemma*. However, analemmatic sundials do not necessarily have a correction for the equation of time built into their design for indicating mean time.

THE LONGWOOD GARDENS SUNDIAL

The Longwood Analemmatic Sundial is 24 by 37 feet in size with morning and afternoon analemmas, along which two gnomons are moved daily. The hour markers are movable to change for daylight saving time. It was built in 1939 based on the design of the historic sundial at the cathedral of Brou in France. Noon observations were made for six years prior to the completion of the sundial. In 1946 du Pont commissioned work on repositioning the hour markers of the sundial. It was originally calculated for 11 am and 1 pm, but not for other hours.

In 1968, Art Jarvela was the director of maintenance for Longwood Gardens. His job description included a long detailed list of the standard type of maintenance that would be expected at a large complex facility such as Longwood Gardens. However, the last line of the job description said “Fix the sundial.” That was one thing he did not know how to do. Hence, he contacted the US Naval Observatory and talked about the problem with Ralph Haupt, Assistant Director of the Nautical Almanac Office. Art and Ralph measured the sundial in detail.[2]

Ralph Haupt presented me with the problem, with some suggestions concerning the computations to be made. I computed a solar ephemeris of the apparent azimuth and altitude for the sundial location for the year 1962, as an average non leap year. Then I computed an hourly location for the gnomon on the analemma and a position between each pair of hours. The positions were very different from the measured positions on the current analemmas. There was no single position that would give the correct time for the morning or afternoon hours. I realized that the true positions for an accurate reading required that the gnomon be moved hourly along a helical path around the analemmas for both the morning and afternoon. At noon the major axis position was most important, and in the early morning or late afternoon the minor axis position was most important. So the x positions were weighted by the cosine of the solar azimuth and the y positions by the sine of the azimuth. Since the number of hours of daylight varies during the year, there is a slight offset in the smooth shape of the analemmas in the spring and fall. The average error for any day is less than one minute. The maximum error during a year is 2.71 minutes for 7 a.m. and 2.12 minutes for noon. The computed analemmas were then checked by determining the errors for all hours of sunlight during the year. Plots of the morning and afternoon analemmas were sent in April 1971 to Art Jarvela along with information concerning what we had done and the expected errors from this design, some details of which are given in the Appendix.

CHECKING THE SUNDIAL DESIGN AND COMPUTATIONS

When the new design and computations had been applied to the sundial, Raynor Duncombe (Director of the Nautical Almanac Office), Ralph Haupt, and I, together with our wives, travelled to Longwood Gardens in the summer of 1971 to check on the results. We arrived the afternoon of the day before we were to meet the people of Longwood Gardens, so we went to see the sundial late that afternoon. We discovered that they had painted the analemma on the surface, so it could be removed if it was wrong. The gnomons were in the correct positions for our arrival the next day. We adjusted them and took readings for an hour or so. Then we placed the gnomons back where they should be for the next day. The next day Art Jarvela and some staff of Longwood Gardens joined us in observing the readings of the sundial for a period of time. It was giving the readings and errors as we had calculated. After years of checking, the analemmas were engraved in stone in 1978.

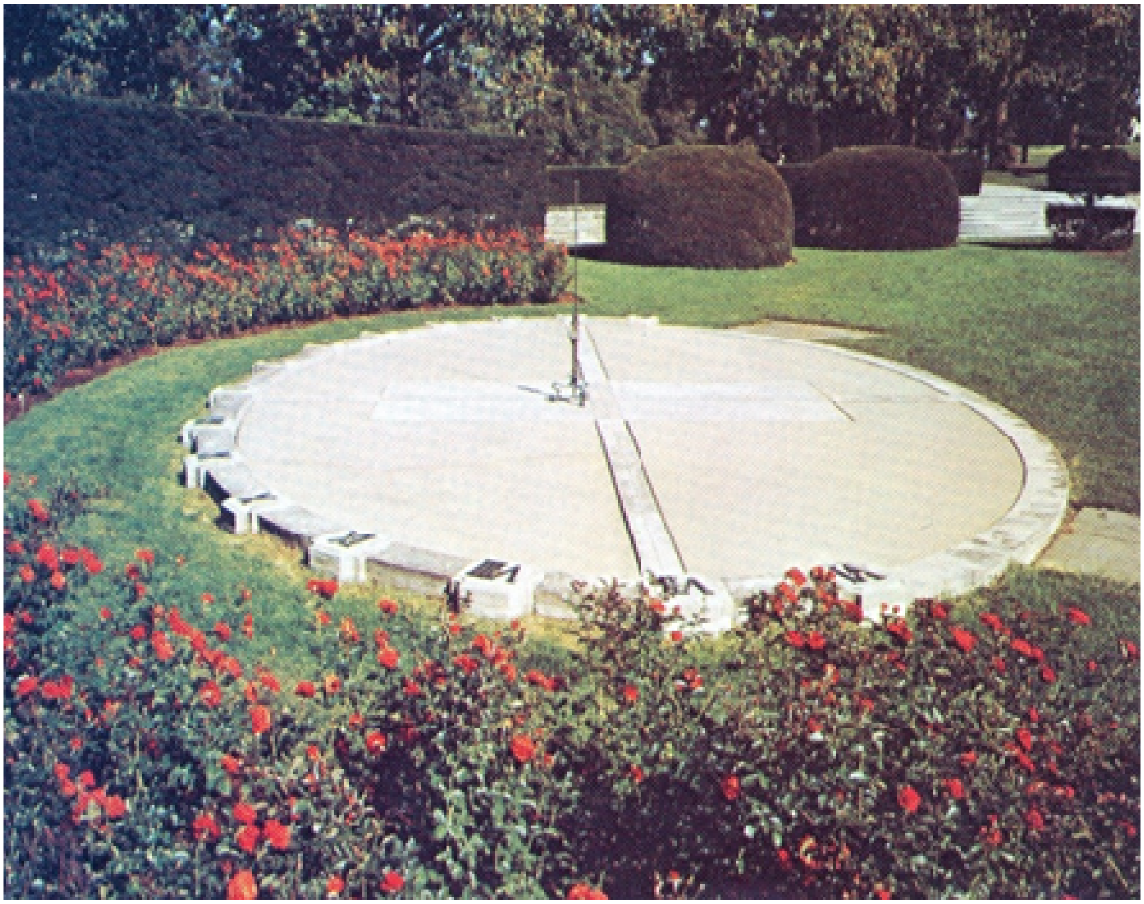

Figure 1. The Analemmatic Sundial at Longwood Gardens.

THE SUNDIAL AT BROU

The Longwood Gardens sundial copied the historic sundial in front of the Cathedral of Brou at Bourg-en-Bresse, France, which was known to have been re-designed by Jerome Lalande in the mid-18th century. I was familiar with the name Lalande, because in 1795 Lalande had recorded two observations of a star that he noticed had moved between the days of the observations. He recorded the observations, but he did not follow up on them. They were observations of Neptune made some 70 years before the discovery of Neptune and with an offset of an unknown source from the current ephemeris of Neptune. Since Lalande was a good astronomer, it was assumed that the sundial in Brou told the correct time. If the Longwood sundial was originally a copy of the Brou sundial, why didn't the Longwood sundial tell the right time?

History of the Sundial in Brou

The methods of projection used to create an analemmatic sundial go back to Vitruvius and Ptolemy.[3] However the analemmatic sundial in Brou may have been the first of its type. There is some uncertainty concerning its construction date, either 1513 or 1532. It was originally built to determine the payments for the workmen building the cathedral. It was located on the cathedral walkways, so people walked on it and wore off the engraved markings indicating where the gnomon should be placed during the year. In 1756 Jerome Lalande rebuilt the sundial and described the theory and difficulties in a memoir in the French Academy of Science in 1758.[4] In 1644 Vanzelard, an expert in analemmatic sundials, described the Brou sundial.[5]

Figure 2. The Sundial at the Cathedral of Brou, France.

Current Sundial in Brou

I contacted the cathedral of Brou seeking an explanation as to why the copy of that sundial in Longwood Gardens did not tell the correct time. A historian at the cathedral responded and informed me that the sundial at Brou did not tell the correct time now either. He explained that Lalande's markings locating the gnomon had since worn away and that an amateur artisan had restored the gnomon markers with one of his own design in 1902. The new feature was that the dates were only located on a surrounding figure-eight analemma, while only the months were on the meridian line. The more precise graduations on the surrounding curve led users to think that the gnomon belonged on the decorative analemma curve instead of on the meridian, which resulted in an inaccurate dial. The historian requested that I send a letter to the cathedral supporting the restoration of the sundial in Brou so that it would tell the correct time as with the Lalande design.

ANALEMMATIC SUNDIALS

Analemmatic sundials have become increasingly common in the 20th century in public spaces, where a person usually acts as the vertical gnomon. Early and more well-known examples of analemmatic sundials also exist in Dijon, Montpellier, and Avignon, France, Vienna, Austria, and Basel, Switzerland. These examples all have the gnomon on the meridian line, the equation of time correction is applied separately. This would be like the original versions of the sundial at Brou. So an amateur artisan designed a sundial that would not tell the correct time. However, the application of computer technology and accurate computations resulted in a correction to that analemmatic sundial design so it would tell mean solar time directly, including the correction for the equation of time.[1]

ACKNOWLEDGEMENTS

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APPENDIX*

General Information

The analemmatic sundial differs from other sundials, because the gnomon, the index casting the shadow, is moved periodically so the sundial will indicate the correct time. This contrasts with a fixed gnomon type where corrections are made to the readings. It will be shown that the accuracy achievable will depend on the frequency with which the gnomon is moved. It is possible that analemmatic sundials are rather old, although the evidence is not convincing. Vitruvius in his book on architecture refers to "people familiar with the analemma" and the theoretical conception of this type of sundial shows similarity to the graphical methods used in works such as "Treatise of the Analemma" by Ptolemy in the second century A.D.[3] One of the oldest and most famous analemmatic sundials in existence is in front of the Cathedral of Brou at Bourg-en-Bresse, France. The sundial dates from the epoch of construction of the Cathedral, about 1506. It was remade by Lalande in 1756, after he conceived the theory of the analemmatic sundial which he published in *Memoires de l'Academie do Sciences* 1757. This sundial has dimensions of 11.18 meters (36.68 feet) by 9.09 meters (29.82 feet) according to René R.J. Rohr in *Les Cadrans solaires* edited by Gauthier-Villars 1965, which is a source of information on various types of sundials. There is a similar sundial in a park in the city of Dijon. The largest analemmatic sundial in the United States is located at Longwood Gardens, Kennett Square, Pennsylvania, and has the dimensions 37.223 feet by 23.832 feet. The layout and calculations for this sundial are the subject of the remainder of this appendix.

Theory of Construction

An analemmatic sundial will have the shape of an ellipse (except at the Earth's poles, where it will be a circle, and along the equator where it will be a straight line). The major axis is oriented East-West while the minor axis is North-South. The ratio of the minor axis to the major axis of the ellipse should equal the sine of the latitude of the location, or

$$\frac{\text{minor axis}}{\text{major axis}} = \sin(\varphi) \tag{1}$$

where φ is the geodetic latitude. Longwood Gardens is located at 39° 52' 22.2" north latitude, so the sine of the latitude is 0.6410858. The ratio of the minor to the major axis is 0.64159, so the layout of the ellipse is quite accurate.*

* This appendix is based on the unpublished typescript "Analemmatic Sundials" by P. Kenneth Seidelmann, c. 1970.

The positions of the hour markers along the ellipse can be determined from the expressions:

$$\begin{aligned} X &= R\sin(AH) \\ Y &= R\sin(\varphi)\cos(AH) \end{aligned} \tag{2}$$

where 2*R* equals the major axis, *AH* is the hour angle of the Sun, *X* is measured along the major axis, and *Y* is measured along the minor axis. For a sundial located on a standard time meridian, one whose longitude is an exact multiple of fifteen degrees, the hour markers are determined for each hour (HR) by evaluating the hour angle of the Sun (*AH*) in degrees from $AH = 15 \times (HR - 12)$ and then determining *X* and *Y* from the above expressions. 'For sundials not located on a standard meridian, the deviation (*DEV*) in degrees from the standard time meridian must be determined from

$$DEV = \text{Longitude of Sundial - Longitude of Standard Time Meridian.} \tag{3}$$

Then the hour angles must be determined from

$$AH = 15 \times (HR - 12) - DEV. \tag{4}$$

when the theoretical positions of the hour markers for Longwood Gardens are thus calculated, they can be compared to the actual measured positions. Table 1 gives the actual positions, the theoretical positions, and the differences in feet. The largest difference is less than one half of an inch.

The Analemma

The analemma (representing the *equation of time*) is defined by Webster to be a graduated scale of the sun's declination and of the equation of time for each day of the year, drawn across the torrid zone on a terrestrial globe.[6] For most sundials, the analemma is shaped like the figure eight and marked for the months of the year to provide a correction due to the equation of time, i.e. the difference between the position of the true Sun and that of the mean Sun which is the basis of our time system.

A point on the analemma can be determined from the equation

$$x - X = (y - Y)\tan(A) \tag{5}$$

where (*x*, *y*) are the coordinates of the point on the analemma, (*X*, *Y*) are the coordinates of the hour marker, and *A* is the azimuth of the Sun at that time. For any particular time there is one equation and two unknowns.

To determine the analemma for Longwood Gardens, an apparent solar ephemeris was prepared (at the time on magnetic tape) with the apparent azimuth *A* and altitude of the Sun for the coordinates, longitude 75° 40' 29.2" W., latitude 39° 52' 22.2" N., for each hour of the year 1962 (selected as an average year midway between leap years), with the beginning of the Besselian year close to the beginning of the calendar year. Using the azimuth of the Sun and the actual hour markers for Longwood Gardens' sundial, a solution was made for the position of the gnomon for

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[*] The dimensions given above for the sundial at Bourg indicate a ratio of minor axis to major axis of 0.81305, which would be for a sundial at latitude 54°24' N. Bourg is located at approximately 46°14' N. latitude, whose sine is 0.72216. Since this is about the ratio of the minor to major axis of the analemmatic sundial in Fig. 94 of the book by Rohr, the dimensions published in that book probably contain a typographic mistake.

each successive pair of equations (5) for the hours 6 a.m. to 6 p.m. In other words, for January l at 6 a.m. EST, the azimuth of the Sun, A_6, and the hour markers for 6 a.m., (X_6, Y_6) were substituted into the equation (5) giving

$$x - X_6 = (y - Y_6)\tan(A_6) \tag{6}$$

and likewise for 7 a.m. giving the expression

$$x - X_7 = (y - Y_7)\tan(A_7) \tag{7}$$

These two equations were solved for x and y to determine the required position of the gnomon for the sundial to read the correct time during that one hour period. This was then done for each successive pair of hours between 6 a.m. and 6 p.m. The process was followed for each day of the year. While the analemmatic sundial will give its most accurate readings if the gnomon is uniquely positioned for each pair of hours, this will complicate the appearance of the analemma and will require someone to move the gnomon each hour. The hourly positions of the gnomon are a helical curve, whose daily average is shaped like the figure 8. Likewise, the curve through the values for any one hour is shaped like the figure 8. This lack of a discrete value satisfying all the equations (5) for a given day prevents the determination from a least squares solution.

Therefore, some attempts were made to determine an average position for the gnomon for each day. A straight average of values for an entire day and for a.m. and p.m. hours separately was attempted along with weighted averages for the entire day. It was found that the most successful solution was to determine a weighted average for an analemma for the a.m., separate from that for the p.m. The Longwood Garden sundial is designed for two analemmas which facilitates the use of a.m. and p.m. averages. Thus, the a.m. and p.m. values were combined separately with the values of the x coordinates weighted by the cosine of the azimuth of the Sun and the y coordinates weighted by the sine of the azimuth.

This method of weighting arises from the geometry of the sundial. At approximately 6 a.m. and 6 p.m. when the shadow is cast along the major X axis, the position of the gnomon along the X axis is relatively unimportant, while its position along the Y axis is very important. Likewise, at noon when the shadow is cast along the minor Y axis, the gnomon position in Y is much less important than its position in X. The hours entering the average were limited to the pairs of hours for which the Sun is above the horizon, specifically 7 a.m. to 5 p.m. If all the hours that the Sun is above the horizon are included in the average, the number of values entering the average varies during the year causing discontinuities in the analemma curve.

With the average values of the analemma calculated for each half day, the azimuth of the hour markers from those positions could be calculated for each hour and compared to the azimuth of the apparent sun. The angular difference, divided by the difference in the azimuth of the Sun between the successive hours, gives the error in the sundial reading for that hour due to the use of the weighted average position for the gnomon. These error values in minutes were evaluated for each hour between 6 a.m. and 6 p.m. The average error for the hours included in the average for any day is less than one minute. The errors for any given time vary periodically during the year, building up to a maximum and then decreasing. The maximum values for times included in the average occurred at 7 in the a.m. curve and noon in the p.m. curve. The maximum for times not included in the averages occurred at 6 in the a.m. curve and 5 in the p.m. curve. The maxima during the year for each of these times are tabulated in Table 2.

Conclusions

The analemmas of the Longwood Gardens sundial represent the average a.m. or p.m. gnomon positions for each day. The exact positions of the gnomon would describe a helical curve around the a.m. and p.m. averages. A weighted average can be used with reasonable accuracy as long as the a.m. and p.m. analemmas are separate. The average error for any day is less than one minute. The maximum error for times included in the average is -2.71 minutes for 7 a.m. and -2.12 minutes for noon.

Table 1. Actual v. Theoretical Positions for the Longwood Gardens Hour Markers (feet)

Hour	Actual Positions		Theoretical Positions		Differences	
	X	*Y*	*X*	*Y*	*X*	*Y*
6	-18.635	-0.125	-18.626	-0.141	*0.009*	*-0.016*
7	-18.057	2.974	-18.048	2.955	*0.009*	*-0.019*
8	-16.230	5.869	-16.241	5.849	*-0.011*	*-0.020*
9	-13.310	8.355	-13.326	8.344	*-0.016*	*-0.011*
10	-9.500	10.298	-9.503	10.271	*-0.003*	*-0.027*
11	-5.030	11.516	-5.033	11.498	*-0.003*	*-0.018*
12	-0.220	11.947	-0.219	11.941	*0.001*	*-0.006*
13	4.602	11.572	4.609	11.571	*0.007*	*-0.001*
14	9.114	10.404	9.123	10.412	*0.009*	*0.008*
15	13.020	8.542	13.016	8.543	*-0.004*	*0.001*
16	16.005	6.084	16.021	6.092	*0.016*	*0.008*
17	17.911	3.210	17.935	3.226	*0.024*	*0.016*
18	18.588	0.120	18.626	0.141	*0.038*	*0.021*

Table 2. Maximum Errors

Date	Maximum Error (min.)			
	6 a.m.	7 a.m.	Noon	5 p.m.
Feb 10	-3.95	-1.80		
Feb 25			-2.12	
Mar 11				-0.65
May 14			0.77	
May 17	2.36	1.33		
May 18				-0.81
Jul 17				-2.30
Jul 21	-4.35	-2.71		
Jul 25			-2.05	
Oct 18	3.55	1.20		
Oct 20			1.79	
Nov 3				1.40

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