SUN DIALS

By Otto Klotz

Having been requested to design the hour lines for the sun dial on the Sussex Street side of the Grey Nuns' Convent here in Ottawa, it may be of interest to write a brief paper on the subject.

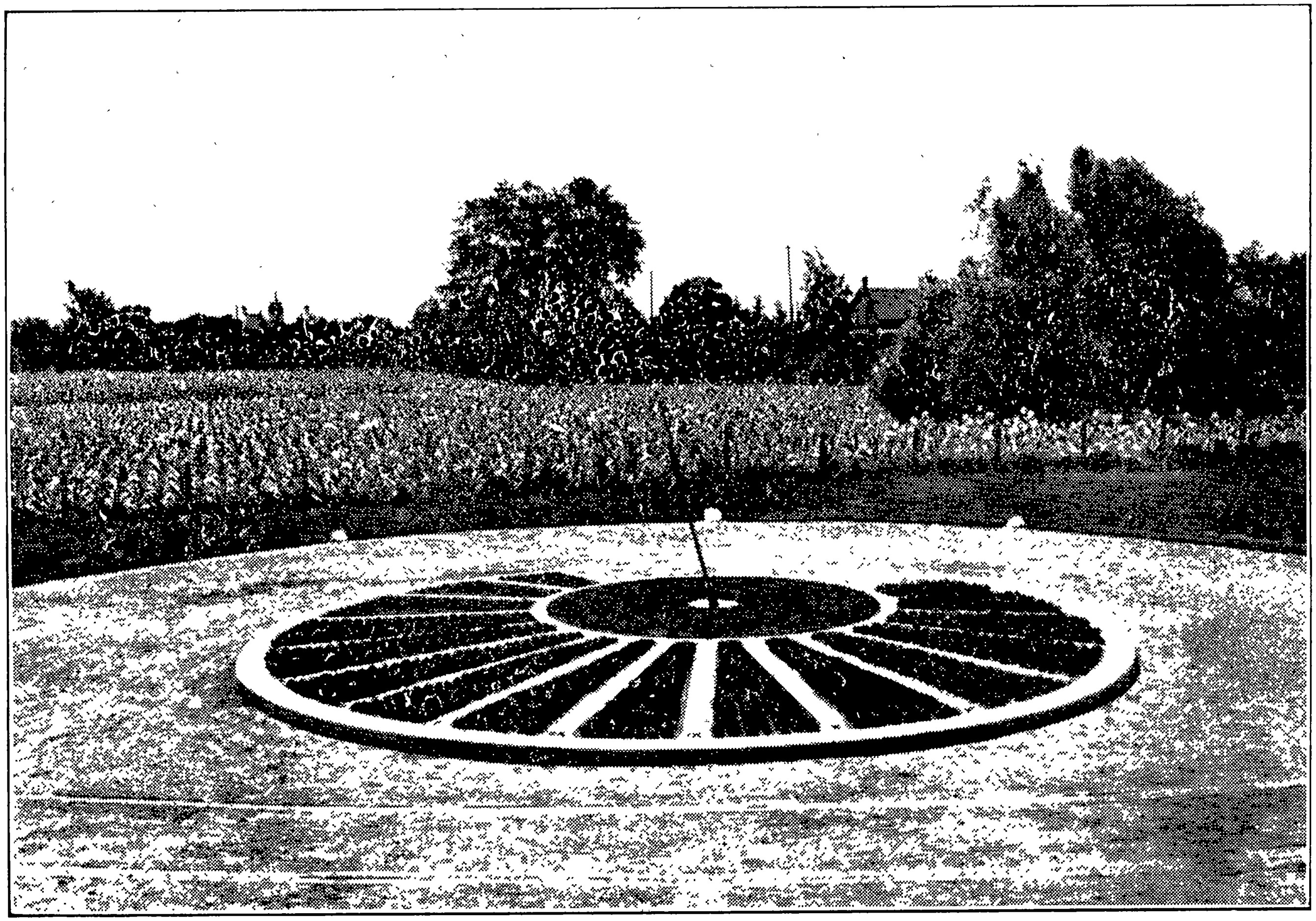
Those that have access to the *Encyclopaedia Britannica*, 11th Edition, will find several pages under the title "Dial", giving the history and the development of dialling or the construction of sun-dials. We will just note that the earliest mention of a sun dial is found in Isaiah xxxviii. 8: "Behold, I will bring again the shadow of the degrees which is gone down in the sun-dial of Ahaz ten degrees backward."

The most common form of dial is the horizontal one. One on a large scale is to be seen at the Observatory here, where the hour lines are separated by flower beds and of which a photograph is herewith given. (Plate VIII).

Let us make clear a few fundamental notions: The sun apparently moves uniformly around the earth, and uniformly whatever the declination of the sun, hence the apparent shadow of the earth's axis moves uniformly in a plane at right angles to it at 15° per hour, or 360°, the complete circle, in 24 hours. Now the sun-dial is simply a modified replica of this. The modification is that the shadow does not fall on a plane at right angles to the axis of the earth but on a horizontal plane, and this horizontal plane makes an angle with the earth's axis equal to the latitude of the place. We could make a dial with a rod and a disk at right angles to it, with the rod pointing to the north pole, thus representing the axis of the earth. On such a disk the shadow would be seen to travel uniformly from one hour line to the next, and so on. But such a disk is inconvenient for mounting and would require graduation on the lower and upper sides for south and north declination of the sun; so recourse is had to a horizontal plane, and the problem is simply to transfer the hour lines from the disk to the horizon, which is readily done graphically or mathematically.

The rod, or stylus or gnomon, is placed in the plane of the meridian and at an angle with the horizon equal to the latitude of the place. Obviously when the sun is in the meridian it is 12 o'clock sun time, or solar time or, as astronomers call it, apparent time.

PLATE VIII.



SUN-DIAL AT THE DOMINION OBSERVATORY, OTTAWA.

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Let *PR* (Fig. 1) represent the stylus, ϕ = the latitude, then the line *RO* at right angles to *PR* represents the trace of the plane in which the sun is moving around *R*, each such plane is parallel to the plane of the equator; and the line *EOW* represents the intersection of the above plane (in which lies *RO*) with the horizon. Let us revolve the plane of *RO* about *O* into the horizontal plane where $OR' = OR$, and the circle with *R'O* for radius represents the uniform motion of the sun. Radial lines drawn from *R'* and the angles

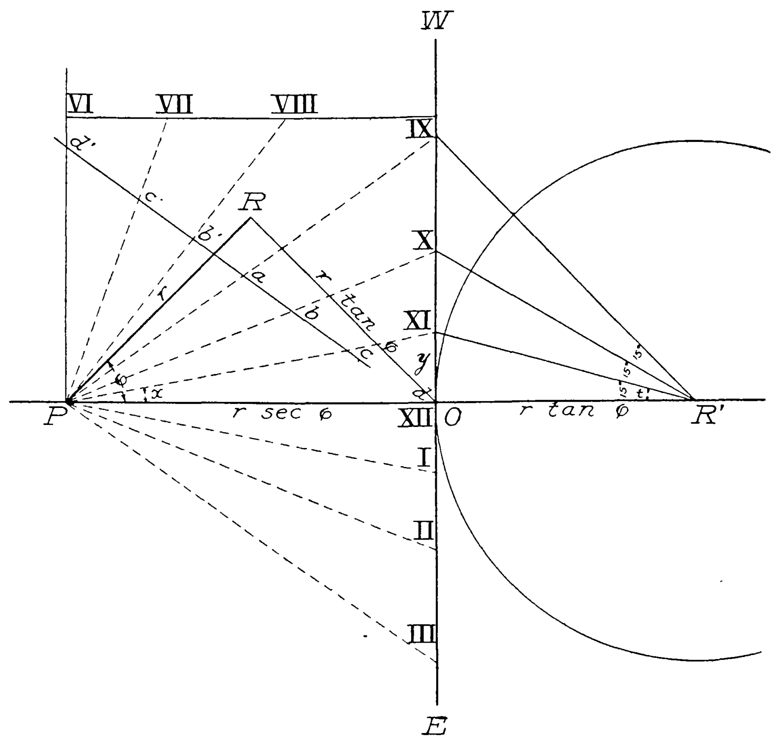

Fig. 1

made with *R'O* the meridian are the hour angles, and their intersection with the line *WOE* will indicate the points where the shadow cuts the horizon. Hence any such point joined to the point *P*, the origin or foot of the stylus, will give the respective hour line for the dial.

The graphical construction is as follows: From *R'* draw lines *R'*XI, *R'*X, *R'*IX, making angles of 15°, 30°, 45° with *R'O*, join *P*XI, *P*X, P*IX*. Beyond IX or III it is generally not convenient to draw the other lines, so we have recourse to a geometrical device (the pretty solution is left to the reader) as follows:

On either the III or IX hour line take any point and draw a line parallel to the other; on this line lay off $ab' = ab$, $ac' = ac$, $ad' = ad$. The last might have been omitted, for in a horizontal dial the VI hour line is always at right angles to the meridian or XII hour line; also the a.m. and p.m. lines of the same hour are in a straight line through the foot of the stylus. So much for the graphical construction; now the mathematics thereof.

Reverting to the figure, let the length of the stylus be r, latitude be ϕ, and hour angle at R' be t, then

$PO = r \sec \phi$, $OR = OR' = r \tan \phi$, $O\mathrm{XI} = y = r \tan \phi \tan t$, and the angle x on the dial for hour angle t is given by

$\tan x = y/r \sec \phi = r \tan \phi \tan t / r \sec \phi = \sin \phi \tan t$. Thus the various values found for x may be laid off for the respective hour lines of the dial.

It may be observed that for the large dial on the observatory grounds the values of y were computed for the five hours and laid off on a tangent line EOW. The points were joined with the foot of the stylus, giving thus the hour lines, which may be enclosed in a rectangular or circular or any other design.

In designing a large dial it is well to remember that the length of the shadow varies considerably during the year, being shortest at the summer solstice. The length of the rod should be such that its shadow at noon reaches to the end of the XII line as laid out on the dial.

Let us next consider a dial on a vertical wall and one running east and west, *i.e.* facing due south. Our diagram is very similar to the preceding one. The rod or stylus again represents the axis of the earth and is placed in the plane of the meridian, making an angle with the vertical wall equal to the complement of the latitude. Hence the angle on the dial representing any hour angle t is given by the relation,

$$\tan x = \cos \phi \tan t.$$

If r is the length of the stylus, its orthogonal projection on the meridian will be $r \operatorname{cosec} \phi$; y will have the same value as before, viz.: $r \sin \phi \cos \phi \tan t$. In the construction of the dial we may lay off y at right angles to the extremity of $r \operatorname{cosec} \phi$ or we may lay off the angles x from the above relation.

We see the construction of the vertical dial is almost identical with that of the horizontal one. The close relationship between the two is well illustrated by a small model readily constructed. Nail two, say foot square, boards together and at right angles to each other. Stretch a stout string from one to the other at an angle to the horizontal board equivalent to the latitude of the place, the plane of the string being at right angles to both boards. Now set the boards so that the string is in the meridian. We have now a horizontal and vertical dial. The point to be brought out by the model is that the hour lines at the intersection of the two boards or horizontal and vertical planes, answer for both dials. The points along this line of intersection designate or indicate the position of the points found from the values of y above.

Now the last case of a dial to consider is the one that gave rise to this short paper; it is the case of a dial on a wall not running east or west, but making an angle (any angle) therewith.

Let the rod or stylus extend from one plane to the other and at the proper angle and in the plane of the meridian. In the above diagram (Fig. 2) P and P' are the extremities of the stylus and WOE the trace of the two planes. Obviously $PO = r \cos \phi$, $P'O = r \sin \phi$; y and x have the same meaning and value as before.

Let SX (Sussex) represent the direction of the wall on which the dial is to be constructed.

It remains to draw the hour lines on the latter wall. Let it be granted that the points for y have been established along WOE as before. Take one of them, α, with $P\alpha$ the particular hour line. This hour line cuts the trace of the wall at a, hence a is a point on the hour line on the wall while P', the extremity of the stylus where the shadows diverge, the other point. The angle θ at P' is given by the relation $\tan \theta = Oa/P'O$.

In the triangle $Oa\alpha$ we have $O\alpha = y = r \sin \phi \cos \phi \tan t$, $\tan x = y/r \cos \phi = \sin \phi \tan t$; the angle $aO\alpha = A$, and $Oa\alpha = 90° - x$, hence Oa or $K = y \cos x / \cos (A \mp x)$, and therefore $\tan \theta = k/r \sin \phi = \cos \phi / \cos A \cot t \pm \sin A \sin \phi$. Hence θ can be computed from this expression.

The graphical solution is very simple. Obtain the intersections a, b, c, d—for the hour angles 1^h, 2^h, 3^h, 4^h—given by the points α, β, γ, δ—for the various values of y. Transfer the distances Oa Ob, Oc—to the WOE line, whereby we obtain a graphical representation of $\tan \theta$ in having the two sides of the right angle triangle $P'O$ I, $P'O$ II, $P'O$ III.

It may be observed that while on a horizontal dial and a vertical one in the prime vertical the six hour line is at right angles to the meridian or XII hour line, this is not the case when the vertical wall is not in the prime vertical or an east west line. The intersection f of a line through P parallel to WOE with the line SX gives the distance Of to be laid off on WOE for the point of the afternoon VI hour line. When t exceeds 90° or 6 hours, tan t becomes negative and we measure from O in the opposite direction,

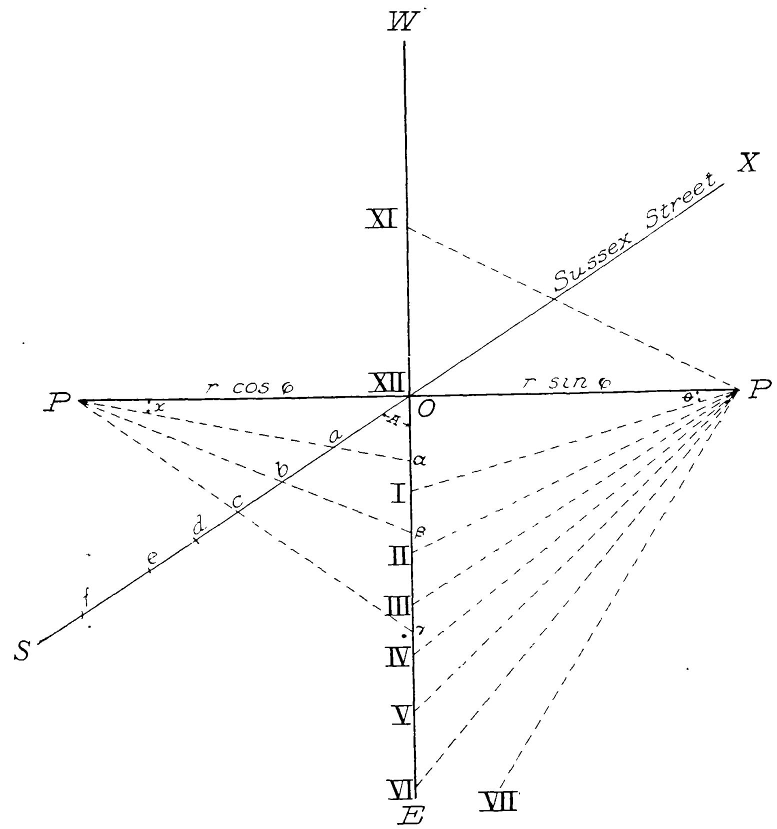

Fig. 2

and obtain an intersection through P with SX, giving us the distance to lay off from O for the hour angle in the above case 7p.m.

A closing word about solar time or the time shown by a sun-dial. Sun-dials at one time served a useful purpose, but to-day are only an ornament, a sentiment, a reminiscence of the days when there were few or no watches. Although the activities of life are regulated by or dependent upon the sun, yet the sun is a very poor time piece. For the purpose of illustration of the apparent shadow of the earth's axis we spoke of the motion of the sun about the earth, of course it is the rotation of the earth that produces sun-rise and sun-set. The days as shown by a sun-dial are not of equal length, although the sun-dial shows the true solar time.

The inequality of the length of the days is due to two causes. In the first place the oribt of the earth about the sun is not exactly circular but eccentric so that the motion is not quite uniform, and furthermore the orbit is inclined to the plane of the equator which introduces another irregularlity in the length of the day. For purposes of life we have adopted days of equal length, that is, averaged up all the solar irregular days, into mean days. This is the time kept by watches and clocks. Four times a year the solar time and mean time are the same; on all other days a correction must be applied to the time shown by a dial to obtain the corresponding mean local watch time. It is perhaps a little too much to expect the man on the street to decipher the relationship between the time shown by a dial and our modern time-keeping. This is what he would have to do. He would have to apply for the particular day the correction, which at a maximum in November amounts to 17 minutes, to dial time to obtain local mean time. It will be observed that the word local is used, a time that is now practically obsolete in the civilized world; all local mean time having been replaced by standard time. The local mean time is the mean time for the particular meridian of the place (of the dial). To this we have to add or subtract the number of minutes and seconds that the place is east or west from the standard meridian—even hours from Greenwich—governing the time of the place. Here in Ottawa our local mean time is slow 2 minutes and 52 seconds on standard time. Having made the above two corrections, if it is summer we make another correction by adding an hour to our result, in order to obtain Daylight Saving Time.

We look at the dial, we look at our watch, and probably feel like saying, "Somebody is lying." At Port Arthur this difference may be over two hours, and the plaintiff would appear to have made out his case.

The sun-dial gives the correct sun time, our watches give the time to suit the convenience of man.

DOMINION OBSERVATORY, OTTAWA, CANADA.
January 21, 1920.