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## They are the shadons that define our every sunny day.

- Sherlock Holmes in Sherlock: The Abominable Bride

[^0]
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# Sundials for Starters - Right Triangles and a Classic Equation Robert L. Kellogg (Potomac MD) 

At this year's NASS conference in Portland, Maine, I was given the opportunity to talk at the Portland Public Library about Mathematics and Sundials. This month I'm going to use part of that talk with some additional material for those deeply interested in dialing.

In most textbooks, Pythagoras of Samos in the 6th century BCE is given credit for discovering the properties of the right triangle (where one of the angles is $90^{\circ}$ ) where the sum of the squares of the sides is equal to the square of the hypotenuse (Fig. 1), or $a^{2}+b^{2}=c^{2}$.


Fig. 1 Pythagoras and a 3-4-5 right triangle


Fig. 2 Plot of Pythagorean Triples (under 4500)

Not only do we ascribe the Pythagorean Theorem to him, but we also acknowledge that he found special right triangles where integer length sides give an integer value for the hypotenuse. Most famous is the triplet $3,4,5$ above. Not only do the integer sides $3,4,5$ form a right triangle, but so do $8,15,17$, and 7,24 , 25. In fact, there are an infinite number of integer triplets that satisfy the Pythagorean Theorem.

Suppose we plot the integer values of $a$ and $b$ that result in an integer value of $c$. The result (out to 4500) is a most amazing plot shown in Fig. 2.

We can certainly appreciate some of the dominant straight lines arising through simple multiplication of smaller "magic" triplet sides. For example $3,4,5$ multiplied by two gives $6,8,10$, and multiplied by three gives $9,12,15$ and so forth. The $a$ and $b$ of course can be reversed, giving rise to symmetric points in Fig. 2 such as $(3,4),(4,3),(6,8),(8,6)$. But the mathematics is more interesting than that. A closer look at Fig. 2 reveals a variety of arcing lines that while not continuous, are certainly not random. This is pretty interesting stuff based only on a simple integer sided triangle.

Was Pythagoras really the first discoverer of these integer triangle triplets? Plutarch states that this relationship was known by the "Ancient Egyptians". But there is proof that this relationship was known much earlier.

In the desert of Iraq nearly 70 years ago a peculiar cuneiform clay tablet was discovered. Now called Plimpton 322 and located at Columbia University, the tablet contains columns of numbers that by their style date to around 1800 BCE. ${ }^{1}$ (Fig.3)


Fig. 3 Babylonian Tablet of Triplets - Plimpton 322

The Babylonians used cuneiform writing with a base 60 numbering system. O. Neugebauer and A. Sachs deciphered Plimpton 322 as a table of right triangle triplets listing a column (2) of triangle diagonals, a column (3) of corresponding integer short side of the triangle, and a column (1) that contains the squared ratio of the diagonal to the second longer triangle side. Here are several examples from the tablet. For a complete listing, see Ref. 1 and discover the errors that the Babylonian mathematician (or copying assistant) made.

| Col. 3 - Width | Col. 2 - Diagonal | Implied Remaining <br> Side | Col. 1 - Ratio |
| :---: | :---: | :---: | :---: |
| 119 | 169 | 120 | $1.98340 \ldots$ |
| 4601 | 6649 | 4800 | $1.91880 \ldots$ |
| 65 | 97 | 72 | $1.81501 \ldots$ |
| 319 | 481 | 360 | $1.88519 \ldots$ |

Apparently the Babylonian mathematician was building a table of trigonometric values, but the reason for it remains unknown.


Fig. 4 Triangles add strength to a bridge.
Why do most of us care about triangles? I use the 3,4,5 right triangle to lay out square chalk lines for laying tile on the floor or cutting a large panel of plywood exactly square. Triangles add structural strength to bridges, buildings, roofs... almost everything. (Fig. 4) But in making those triangles, we always need to know their precise lengths. Consider my college experience with triangles:

There was a time in college when I heard about the Buckminster Fuller Dome and I wanted to build one. On a summer's evening I started with a box of toothpicks and began gluing together my dome. I could

[^1]assemble a triangle made of the first, but flat tessellation, but when I began fitting the triangle pieces together into a dome, they wouldn't fit together. I worked into the early hours of the morning with no success. Years later I returned to the problem and realized that to succeed in making a Buckminster Fuller Dome, the tessellations of a dodecahedron are not all the same length. That's why the climbing dome for kids is usually color coded so that at least there is a chance of assembling it correctly. (Fig. 5)


Fig. 5 Geodesic Dome and Tessellations
The utility of triangles was instilled in me at an early age by my father, a carpenter. I accompanied him during the summer to construction sites and watched as he laid out the roof rafters of a house using a simple carpenter square. The ceiling joist is the run and the roof height is called the rise. (Fig. 6) The slope of the rafter is given in proportions of rise to run such as $5: 12$, called the pitch of the roof (and you can guess that the ratios were measurements in inches). Thus there is a gamut of different American roofs. Take a look at Fig. 7 and the different pitches of common roofs.


Fig. 6 Rise and Run of a roof
Returning to the humble triangle of rise and run with a ratio called pitch, we can immediately declare "Eureka", we've been doing trigonometry. In fact, Fig. 8 shows that we can write:

$$
\tan \theta=\text { pitch }=\frac{\text { rise }}{\text { run }}=\frac{b}{a}
$$



Fig. 7 Pitch of house roofs


Fig. 8 Rise, Run, Pitch and the Tangent
And we have the other trigonometric identities too:

$$
\sin \theta=\frac{b}{c} \quad \text { and } \quad \cos \theta=\frac{a}{c}
$$

We can turn these equations inside out, where if we have the angle, we can compute the ratio of the sides of the right triangle (using modern calculators of course rather than scribing cuneiform numbers in the Babylonian sexagesimal number system):

$$
\theta=\tan ^{-1}\left(\frac{b}{a}\right) \quad \theta=\sin ^{-1}\left(\frac{b}{c}\right) \quad \theta=\cos ^{-1}\left(\frac{a}{c}\right)
$$

Now that we have the basics of triangles, I'd like to attempt a simple and visual way of deriving the classic trigonometric equation for laying out the hour lines on a horizontal sundial:

$$
\tan \theta=\sin \phi \tan H
$$

where $\theta=$ sundial hour line angle from noon
$\phi=$ sundial site latitude
$\mathrm{H}=$ sun hour angle ( $+/-$ ) from the local meridian

I'm inspired by Jack Aubert who several years ago sent me an email with a photo of his trigonometric model explanation saying, "Actually, I'm one of the last people who should be attempting this math. I think the reason I would like to see some more of it is that I find it very challenging and need some visual aids to understand it." So with that, here we go.


Fig. 9 Erickson Equatorial Dial in Littleton, Colorado
I've always been intrigued by the Erickson Monument equatorial dials and they are ideal for showing how to derive the fundamental horizontal shadow equation. First, let's look at the Erickson dial from Littleton, Colorado (Fig. 9). The axis is pointed to the North Celestial Pole, making an angle to the ground equal to the site's latitude. The equatorial disk shows the hours from local solar noon. The gnomon casts a shadow from spring to fall equinox on the dial's front while the back side receives the axis shadow from fall to spring. (One can see why this is an unpopular dial at very high latitudes.)
Fig. 10 shows my simple model of the equatorial dial, located at latitude $\phi$. The morning summer sun shines on the gnomon, creating a shadow at approximately 10 am . The line NS is a north-south line from the base of the equatorial plate to the foot of the polar axis. The EW line is an east-west line just touching the equatorial plate.

We construct three different triangles on this equatorial dial (Fig, 11). (a) the latitude triangle NOS that includes the latitude $\phi$ of the polar axis (site latitude); (b) the hour angle triangle NOW that includes the shadow hour angle H of the sun; and (c) the hour line triangle NWS that includes the hour line angle $\theta$. This triangle is horizontal, and the hypotenuse represents the shadow of the polar at hour angle H .
The three triangles create the following simple trigonometric relationships

$$
\frac{N O}{N S}=\sin \phi \quad \frac{N W}{N S}=\tan \theta \quad \frac{N W}{N O}=\tan H
$$

Rewriting the latitude and hour line equations gives us the length of the line NO and NW with respect to the baseline NS and the angles of latitude $\phi$ and hour line $\theta$ :

$$
N O=N S \sin \phi \quad N W=N S \tan \theta
$$

Substituting these relations into the equation for hour angle gives:

$$
\frac{N S \tan \theta}{N S \sin \phi}=\tan H \quad \text { or } \quad \frac{\tan \theta}{\sin \phi}=\tan H, \text { rewritten as } \tan \theta=\sin \phi \tan H
$$



Fig. 10 Equatorial Dial model


Fig. 11 Construct three triangles based on the dial
This last equation is indeed the classic relationship between hour line angle $\theta$, latitude $\phi$ and hour angle H of the sun. To actually compute the resulting hour angle in a less memorable and less elegant form, we use:

$$
\theta=\tan ^{-1}(\sin \phi \tan H)
$$

All done with simple right triangles. Now go use the equation to draw your own horizontal sundial knowing that there is a lot of history behind it.

## Globe Sundial of Prosymna - The Enigma Of Its Ancient Construction Is Solved Ortwin Feustel (Glashütten, Germany)

Mathematical analyses [4-6] of the circa 2000-year-old Globe Sundial of Prosymna, Fig. 1, have brought to light, above all, the profound astronomical knowledge of its creators: time of day and season are indicated exclusively by means of the terminator, i.e. at the boundary line between the illuminated and nonilluminated halves of the globe. Until now it has been a mystery how the sundial's design was done, and with such exactness but without formulae!

The following discussion presents a logical concept that tries to answer the question, it provides the construction in two dimensions (drawings) with a pair of compasses and ruler, and in three dimensions with the help of auxiliary mechanical equipment. 3D and 2D representations offer step by step visualization of the design procedure.

## Functions of a row of holes and three line nets [4-6]

The purpose of the holes (mounting positions for metal knobs) was clearly first and foremost to indicate the temporal hours at the solstices, i.e. the intersection of the terminators at the solstices gives the exact hole positions. But due to the insignificant deviations for the terminators for non-solstitial declinations it is possible to read the temporal hours during the whole year.
The nature of a spatial curve and its position on the sphere's surface determine shape and function of each line net, see Fig. 2. In the course of a day the terminator plane is the tangential plane to the sheet of a double cone; the half opening angle of the cone equals the daily sun's declination. The terminator's great circle cuts all algebraic curves. By the way, the meridian is also the line for the temporal hours zero and twelve.
Line net I consists of two branches of a spatial Pascal's limaçon (a conchoid of a circle) as solstitial declination lines, an equinoctial circle and ten pairs of great circle arcs arranged between the algebraic curves; these arcs - looking like a herringbone - are the hour lines. Evaluating the terminator position in line net II also yields the relevant time and season sought.

Line net II consists of one great circle arc (equator), six parallel circle arcs according to the zodiacal declination values and twelve temporal hour lines. Evaluating the terminator position in line net I also yields the relevant time and season sought.

Line net III with three great circle arcs, including the horizon, and twenty-two small great circle arcs (herringbone) serves solely to indicate the equinoxes.


Fig. 1: Northern view of the Prosymna globe with complete net I (spherical Pascal's limaçon = declination lines, equinoctial circle, ten hour lines $=$ herringbone, meridian), parts of net II (three declination lines, nine hour lines) and eight holes including zenith. Photo: Karlheinz Schaldach


Fig. 2: 3D representation with the red colored declination lines of net $I$, blue colored declination lines of net II and cyan and green colored half circles, respectively, of net III.

## Ancient fundamentals for the globe dial's design

Without knowledge of at least the following, it would be impossible to create the unique Sundial of Prosymna:

- the Analemma of Vitruvius ${ }^{1}$, see Fig. 3 and [1-3], for finding
- the sun's declination, depending on the ecliptical longitude according to the individual zodiacal signs,
- daylight length and therefore the angle of one temporal hour, if geographical latitude and sun's declination are given;
- astronomical laws such as
- the obliquity of the ecliptic equals the difference of the sun's noon altitude at the summer and winter solstice divided by two [3],
- the geographical latitude equals the altitude of the celestial pole measured from the horizon and results also from the equinoctial culmination altitude subtracted from $90^{\circ}$ [3],
- the so-called Indian Circle delivers the north-south direction [3],
- the daily course of sunrays forms a cone with the vertex touching the top of a gnomon,

[^2]- the sunray is normal to the terminator plane;
- projection of a body on three planes;
- penetration of bodies;
- penetration of a double cone and a hemisphere,
- projection of the penetration line onto a special plane (e.g. the cone's base plane).


Fig. 3: The Analemma of Vitruvius [2]. $\mathrm{AB}=$ gnomon, $\mathrm{BC}=$ shadow cast at the equinoxes, $\mathrm{AC}=$ sunray at the equinoxes, $\mathrm{EI}=$ horizon, Menaeus $=$ circle around $\mathrm{D}, \mathrm{NF}=$ equator, $\mathrm{LH}=$ ecliptic.

We can and must assume this minimum knowledge. Finally there is no doubt about the imaginativeness and craftsmanship of the tradespeople and stonemasons.

## Seasonal sun's declination and temporal hour angle

The Analemma of Vitruvius, Fig. 4, is a two-dimensional projective drawing of the celestial sphere. The lines parallel to the equator correspond to the appropriate parallel circles.

## Seasonal sun's declination

The seasonal sun's declination values are causally related to the ecliptical longitude of the zodiacal signs. For this reason the Menaeus circle is placed between the intersection points of meridian and solstitial lines (red and blue colored, respectively) so that the connecting line of $\lambda=0^{\circ}$ and $\lambda=180^{\circ}$ (equinoxes) is aligned with the equator; therefore the following assignments result: $\lambda=30^{\circ}$ and $\lambda=150^{\circ} \Rightarrow \delta=11.5^{\circ}, \lambda=60^{\circ}$ and $\lambda=120^{\circ} \Rightarrow \delta=20.2^{\circ}$ and so on.

## Length of one temporal hour

With $\varphi=35.7^{\circ}$ and $\delta=11.5^{\circ}$ as an example, the half daylight arc OA- 6 divided by six yields the length of a temporal hour $\tau_{\mathrm{c}}=98.4^{\circ} / 6=16.4^{\circ}$.


Fig. 4: Vitruvius‘ Analemma for the construction of declination values and temporal hour length according to the ecliptical longitude at the start of each zodiacal sign. Symbols used are: $\varepsilon=$ obliquity of the ecliptic, $\lambda=$ ecliptical longitude, $\phi=$ geographical latitude, $\delta=$ sun's declination, $\tau_{\mathrm{c}}=$ angle of one temporal hour, $\mathrm{K}=$ parallel circle's center, $\mathrm{H}=$ intersection of declination circle (parallel circle) and horizon - point H appears rotated into the meridian plane as point A , half daylight arc = eastern parallel circle arc rotated into the meridian plane, 0 to $6=$ count of the temporal hours from sunrise to noon, $\mathrm{O}=$ globe's center, $\mathrm{NP}=$ north pole, $\mathrm{S}=$ south.

## Construction of Pascal's limaçon

Figs. 5 and 6 show the positioning of a double cone at the top of a sphere and how the double cone penetrates the sphere. The penetration lines are two spatial curves that are similar to circles, but the true character of this pair of curves is not discernible at first. Fig. 7 reveals that the projection of the greater curve onto the cone's base plane is the outer branch of a Pascal's limaçon; consequently the smaller curve is the limaçon's inner branch. Fig. 8 completes the 3D representations with a meridional section of Fig. 10.


Fig. 5: 3D representation of two cones positioned with their vertices at the zenith of a sphere; the cone sheets have been drawn only as far as the intersection with the meridian $w$ and s, respectively. See also Fig. 6.


Fig. 6: Penetration of a double cone and a hemisphere; the vertices of the cones are positioned at the hemisphere's zenith. The intersection of both surfaces - cone and the hemisphere - forms a spherical Pascal's limaçon.


Fig. 7: Penetration of a cone (summer solstice cone) and a sphere as well as the projection of the penetration line onto the cone's base plane (it is parallel to the equinoctial plane) with the help of the spherical surface method [7]; the projected curve is the outer branch of a Pascal's limaçon. Symbols used are: $\mathrm{e}=$ intersection point of the equinoctial plane with the globe's meridian, $s=$ intersection point of the cone's base plane with the globe's meridian, $\mathrm{S}=$ orthogonal projected point s onto the equinoctial plane $=$ vertex of the outer branch of a Pascal's limaçon.


Fig. 8: Meridional section (yz plane of a right-handed system) through the 3D representation of Fig. 9. The cone sheets (conchoid's branches) intersect the meridian at the points $w=$ winter solstice and $s$ = summer solstice; the equinoctial circle cuts the meridian at e. Further symbols and their meanings are: $\phi=$ geographical latitude, $\delta=$ solstitial declination, $\mathrm{O}=$ origin of the coordinate system, $Z=$ sphere's zenith, $\mathrm{NP}=$ north pole, $\mathrm{SP}=$ south pole, $\mathrm{S}=$ south.


Fig. 9: The sun's vector is the normal vector of the terminator plane; the second temporal (equinoctial) hour is shown by the sunray touching the equator and the terminator plane cutting both the equinoctial circle of net I at point $\mathrm{c}=2$ and the meridian.


Fig. 10: View into the globe's interior; the solstitial double cone intersects the meridian at w (blue colored winter cone) and s (red colored summer cone), the inserted terminator for the sixth temporal hour at the winter solstice is tangent plane to the double cone and cuts the meridian at w , the black equinoctial circle cuts the meridian at e and coincides at the same point with the equinoctial double cone collapsed to a straight line. See also fig. 6.

## Simulation of the terminator

Figs. 9 and 10 demonstrate the interdependence of sunray and terminator, as well as the interplay between the two double cones and the terminator. Both figures suggest a mechanical device that takes account of the fundamental sunray-terminator principle, namely that the sunray always passes through the zenith of the illuminated half of the globe. Fig. 11 shows a possible realization of such a simulator, for instance made of rigid wire.


Fig. 11: Mechanical counterpart to the combination of terminator plane and sunray. The grey half-ring with the red colored 'eye' is perpendicular to the red terminator ring; the ring's radii equal the globe's radius. $\mathrm{O}=$ center of the terminator plane $=$ globe's center, P corresponds with the individual temporal hour positions on the declination lines of net II.

## Auxiliary Equipment

The pair of holes shown in Fig. 12 served - with greatest probability - for affixing an auxiliary tool with two small rods. Such a tool is suitable for the precise positioning of a stencil with holes in order to transfer the Pascal's limaçon - as shown in Fig. 7 - point by point onto the globe's surface. Fig. 13 explains how the support can be applied.

Note: Fig. 11 in [4] and Fig. 6 in [5] mistakenly show the pair of holes at the intersection of meridian and equinoctial circle; however Fig. 12 in [6] is correct.

## Transferring the hole positions and line nets onto the sphere's surface

Preparatory tasks

- Determining
- globe's radius
- geographical latitude
- angle between the great circle arcs of net III
- Construction
- seasonal declination values
- seasonal temporal hour angles as well as their chords
- projective drawing of Pascal's limaçon
- Making
- terminator simulator
- support
- hole stencil with the Pascal's limaçon


Fig. 12: The pair of holes at the intersection point of meridian and horizon circle on the globe's northern half. Photo: Alexander Jones.


Fig. 13: Support for the firm adjustment of a stencil with holes. The angle between the lines Zw and Zs equals the solstitial declination. Symbols used are: $\phi=$ geographical latitude, $\delta=$ solstitial declination, $\mathrm{O}=$ globe's center, $\mathrm{w}, \mathrm{e}, \mathrm{s}=$ intersection points of the spherical declination lines with meridian, $\mathrm{Z}=$ globe's zenith. Only part of the meridional section through globe is shown.

## Stonemason's preliminary activities

- Globe
- Great circles for horizon, meridian and equator
- Two circle arcs for net III
- Six parallel circles for net II
- Equinoctial circle for net I
- Markings for w and s at the meridian
- Hole for zenith and two holes for the support's fastening
- Pascal's limaçon for net I

Transfer of the hole pattern can be done best by means of orthogonal illumination, as it is then possible to mark the positions of projected light spots on the globe's surface. The stencil is placed correctly on the support if the light spots at W and S coincide with the markings w and s at the meridian.

## Usage of the terminator

## simulator

The terminator simulator is central to marking the positions of holes, see Fig. 14, the herringbone of net I, see Fig. 15, the hour lines of net II, see Fig. 16, and the herringbone of net III, see Fig. 17. Fig. 18 illustrates how to proceed on the basis of the third equinoctial and solstitial temporal hour. If the "eye" of the simulator is placed at the point P3e then its "ring" (great circle) cuts the equator of net II, the equinoctial circle of net I (see also Fig. 19) as well as the great circle arcs of net III so that the appropriate intersection points can be marked. Placing the eye at the points P3w and P3s yield the third hole position and the intersection points with the solstitial declination lines of nets I and II.

## Stonemason's final

 activitiesChiseling the holes for the metal knobs and the connection lines between the matching intersection points completes the sundial.


Fig. 14: 3D representation of net II with seven declination lines and one line for the second temporal hour as well as three terminators and holes 0 to 5 . Color red $=$ summer solstice, color blue $=$ winter solstice, color magenta $=$ equinoxes, color cyan $=$ sun's declination $\pm 11.5^{\circ}$ and $\pm 20.2^{\circ}$, color black $=$ hour line and meridian, color green $=$ horizon.


Fig. 15: The drawing of the complete net I (= Pascal's limaçon and equinoctial circle including the herringbone) corresponds to the projection from the top onto the xy plane.


Fig. 16: 3D representation of net II with seven declination lines (blue colored) and hour lines 0 to 6 (black colored) as well as six holes 0 to 6 and the horizon circle.


Fig. 17: The drawing of the western herringbone of net III corresponds to the projection from the south onto the xz plane. In addition, the equinoctial terminator tracks 0 to 6 are shown; $\mathrm{c}=$ hour count.


Fig. 18: 3D representation of net I and net II with solstitial and equinoctial declination lines as well as two solstitial and one equinoctial terminators, each for the third temporal hour. The solstitial terminators are shown as planes; the equinoctial terminator is shown as a great circle; it is shown running only up to the equinoctial circle. Winter and summer terminators intersect at hole position three. The relevant intersection points for net I are: winter terminator $\Leftrightarrow$ conchoid's inner branch, equinoctial terminator $\Leftrightarrow$ equinoctial circle, summer terminator $\Leftrightarrow$ conchoid's outer branch. The relevant intersection points for net II are: winter terminator $\Leftrightarrow$ winter solstitial declination line, equinoctial terminator $\Leftrightarrow$ equator, summer terminator $\Leftrightarrow$ summer solstitial declination line. The direction of sight is at an angle of about $45^{\circ}$ from the south.


Fig. 19: All equinoctial terminators for the hours 0 (= meridian) to 6 are shown as arcs; they intersect each the equinoctial circle and run all through the intersection point of equinoctial circle and meridian. The two holes at the intersection point of meridian and horizon (see Fig. 12) are necessary for fixing on the support (see Fig. 13).

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## Linear Equant Sundial Design <br> Fred Sawyer (Manchester CT)

The first original idea I published in dialing was the Equant Sundial design, developed in the mid-70's but published in 1991 (An Equant Sundial, BSS Bulletin, July 1991, 91(2):34. Reprinted in Sawyer, Sciatheric Notes $-I, 1998$ ). The basic idea is to produce a horizontal dial with a curve on its face that lies over a rotatable array of equi-spaced hourlines centered on a point on the meridian different from the traditional dial center. The hourlines are rotated to easily account for corrections such as the equation of time, and mean time is read by noting the hourline at the point where the customary gnomon's shadow intersects the equant curve (Fig. 1). There is some degree of freedom in the choice of parameters for the dial, so the curve may vary somewhat, but it is generally cardioid in shape (Fig. 2). Bill Gottesman has made several of these dials and he offers a nice graphic explanation of the design on his website at http://precisionsundials.com/why\ it\ works.htm.


Fig. 1. Equant dial by Ron Anthony



Fig. 2. Equant dial by Bill Gottesman


Fig. 3. Example Equant curves using a wide gnomon.
I subsequently returned to this design idea on multiple occasions; see for example: A Bipolar Azimuthal Equant Sundial (Jun 2005), A Self-Orienting Latitude-Independent Analemmatic Equant Dial Design (Dec 2003), Yabashi Point and Sawyer Equant (Dec 2000). In the article on the Yabashi Point, I considered Tokutaro Yabashi's attempt to obtain a similar result: he uses a traditional layout of hour points (on a circle) that of course are not equi-spaced and then introduces error into the design by treating the points as though they in fact are equi-spaced. His goal of course was to minimize the error. My approach to the design was
to avoid introducing error, using hour points that are both exactly correct and exactly equi-spaced but that lie on an equant curve rather than a circle. This generalization allows for the introduction of a wide gnomon and a greater diversity in the look of the equant curve; see for example the equant curves in Fig. 3.

My goal in this article (based on a 2009 talk titled Equants, Scales \& Castillon Tables) is to introduce a new layout that uses only a straight line for the equant - replacing the rather complicated curves.
Begin with an east-west line (Fig. 4); from its center, draw a north line with unit length. On either side of the center, at unit distances, draw two
 more north lines, each of length $2 /(1-\sin \varphi)$, where $\varphi$ is the latitude of the horizontal dial's location.

This dial must be supplied with a traditional polar-oriented gnomon with a width of two units, just sufficient to fill the gap created by the two long north lines.

Fig. 4


Fig. 5


Hour Lines at $15^{\circ}$ Intervals
Fig. 7

Fig. 6


Fig. 8

The hourlines for this dial must be drawn on a rotatable disk with center at the end point of the first (short) north line (Fig. 5). From this center draw lines through the end-points of each of the two longer north lines, extending to the circumference of the hour disk (Fig. 6). These two lines will each represent noon, one for each side of the wide gnomon. Now, from the same center, draw morning and afternoon hourlines from these noonlines - each hour at a $15^{\circ}$ interval from its


To adjust for the Equation of Time, Simply rotate the hour circle around its center (without moving the gnomon or equant line).

Fig. 9 predecessor and successor (Fig. 7).

Finally, note where the right noonline cuts the north line and where the 6 pm line cuts the east-west line; join these two points with a straight line. This is the equant line for the afternoon. Follow a similar procedure to obtain the equant line for the morning (Fig. 8).
Now read the time by noting where the gnomon's shadow crosses the appropriate equant line (Fig. 9). To adjust for the Equation of Time, simply rotate the hour circle around its center (without moving the gnomon or equant line).

I believe this is a simple, interesting, and new approach to designing a dial and adjusting for the equation of time. However, we must now consider how to prove that it works!

## Proof

Consider the afternoon portion of the dial; let the origin of your coordinate system be the point where the east side of the wide gnomon touches the east-west line. The gnomon's shadow will obviously emanate from this point. The equations of the gnomon's shadow and of the afternoon equant line, respectively, are:

$$
x=y \tan t \sin \varphi \text { and } y=\frac{2}{1-\sin \varphi}-\frac{x}{\sin \varphi} .
$$



Fig. 10

The coordinates of the point where these lines cross (the green circle in Fig. 9) are:

$$
x_{t}=\frac{2 \sin t \sin \varphi}{(1-\sin \varphi)(\cos t+\sin t)}
$$

$$
y_{t}=\frac{2 \cos t}{(1-\sin \varphi)(\cos t+\sin t)} .
$$

Now consider angles $\theta$ and $\alpha$ (Fig. 10) whose values are given by the following equations:

$$
\begin{gathered}
\tan \theta=\frac{x_{t}+1}{y_{t}-1} \\
\tan \alpha=\left(\frac{2}{1-\sin \varphi}-1\right)^{-1}=\frac{1-\sin \varphi}{1+\sin \varphi} .
\end{gathered}
$$

The angle $\theta$ is measured at the center of the hour disk, between the extended original north line and the line joining this center and the intersection of the shadow and the equant line. The angle $\alpha$ is between the
extended original north line and the line from the center of the hour disk to the end point of the right-most north line.

Our challenge is to evaluate the angle $\theta-\alpha$. This is the angle between the noon hourline and the hourline selected by the gnomon's shadow as it crosses the equant line. Our hope of course is that $\theta-\alpha=t$, i.e. the angle exactly equals the sun's hour angle - so hourlines equally spaced at $15^{\circ} / \mathrm{hr}$. are appropriate.

Combining the information given above, we have:

$$
\begin{aligned}
\tan \theta & =\frac{x_{t}+1}{y_{t}-1}=\frac{2 \sin t \sin \varphi+(1-\sin \varphi)(\cos t+\sin t)}{2 \cos t-(1-\sin \varphi)(\cos t+\sin t)}=\frac{\cos t+\sin t+\sin \varphi(\sin t-\cos t)}{\cos t-\sin t+\sin \varphi(\sin t+\cos t)} \\
& =\frac{1-\sin \varphi+(1+\sin \varphi) \tan t}{1+\sin \varphi-(1-\sin \varphi) \tan t}=\frac{\frac{1-\sin \varphi}{1+\sin \varphi}+\tan t}{1-\frac{1-\sin \varphi}{1+\sin \varphi} \tan t}=\frac{\tan \alpha+\tan t}{1-\tan \alpha \tan t}=\tan (\alpha+t)
\end{aligned}
$$

So, $\theta=\alpha+t$, or equivalently, $\theta-\alpha=t$. Our equant therefore works correctly.

## Second Proof

Of course, this proof has not been very intuitive or instructive. Let us consider an alternative geometrical approach.

Consider Figure 11. Note that our equant is a straight line fitting exactly 6 hours between the two axes


Fig. 11 (north-south and east-west) - exactly the situation we have with the traditional dialing scales (invented by Samuel Foster!) and representing the easiest way to lay out the hourlines of a horizontal sundial. Could this equant simply be an example of a dialing scale incorporated into the face of the sundial itself?

In Figure 11 we see the equant as the base of a triangle that includes a $90^{\circ}$ angle at the center of the hour disk. Now we ask if the two sides of this right triangle are equal in length.
Referring to Figure 12, we introduce the auxiliary right triangles ADF and CEF with bases DS and EF equal to the unit length we originally adopted for this sundial. From the construction, angle DFA plus angle AFE equals angle EFC plus angle AFE, which equals $90^{\circ}$. So angles DFA and EFC must be equal. We therefore have two right triangles (ADF and CEF) that have equal sides (DF and EF) and equal acute angles (DFA and EFC) - and therefore congruent to each other. Since these triangles are congruent, their hypotenuses FC and FA are equal to each other. And since FC and FA are the two sides of the right triangle we are examining, we can conclude that the triangle is an isosceles right triangle (i.e. it has two equal sides).
Why should this be of interest to us? Because the traditional graphical construction of the hour scale (one of the two usual dialing scales - Fig. 13) shows us that it is simply the base of an isosceles right triangle with hours marked at equal intervals of $15^{\circ}$. And that is exactly what we have here. Our equant line is indeed equivalent to a traditional hour scale, but we are now left with the question of whether or not it is resting in the correct position between the two axes (Fig. 14).


Fig. 12


Fig. 14

Fig. 13


Fig. 15

The key to handling this final hurdle in our proof is to recall that the latitude scale of the traditional pair of dialing scales is designed to assure that, when the hour scale is properly positioned (Fig 15), the angle $\psi$ it makes with the north-south line satisfies the condition: $\tan \psi=\sin \varphi$. So all we need to do is show that the corresponding angle $\psi$ between our new equant and the north-south line satisfies the same condition.

Consider Fig. 16 and recall that the length CB was selected to equal $2 /(1-\sin \varphi)$, the lengths DB and $B E$ are equal to 1 , and, from earlier in our proof, we know that $\mathrm{AD}=\mathrm{CE}$ in the congruent triangles.

With this information, we can follow the proof given in Fig. 16 that indeed we do finally have $\tan \psi=\sin \varphi$. So we not only have a second proof that the equant line works, but we have that proof by showing that the equant design works by embodying Foster's Dialing scales on the face of the dial.


Fig. 16
Fred Sawyer
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# A Graphical Dial Construction By Jean de Castillon Fred Sawyer (Manchester CT) 

My previous article in this issue of The Compendium was inspired by an approach to dial construction outlined in the $18^{\text {th }}$ century by Jean de Castillon.

Castillon's birth name was Giovanni Francesco Melchiore Salvemini de Castiglione; he was born in Tuscany in 1709, studied mathematics at Pisa, and then went to teach in Switzerland, becoming a professor at the University of Utrecht in 1755. In 1764 he moved on to Berlin and was named the Royal Astronomer at the Berlin Observatory the following year. Castillon published the Leibniz - Johann Bernoulli correspondence, edited the works of Euler and published a commentary on Newton's Arithmetica Universalis. He also translated Locke's Elements of Natural Philosophy into French. He was a Fellow of the Royal Societies of London, Gottingen, Harlem \& Berlin and of the Institute of Bologne. In 1784, at the age of 75 , he turned his attention to gnomonics: Sur la gnomonique, Nouveaux mémoires de l'Académie Royale des Sciences et Belles-Lettres, Année 1784, Berlin, 1786, 259-296 + plates V-VII.

Castillon recognized the value of dialing scales for laying out sundials, but he lamented the fact that they were not commonly found in the toolkit of Continental dialists:
The scales of which I am going to speak..., as far as I know, are found only on the English compass of proportion, \& they should be found on all of them.
Rather than mounting a campaign to equip every dialist with a set of dialing scales, Castillon took another approach:
One can do without scales prepared in advance. It is easy, every time one has to trace a dial, to construct scales proportioned to this dial....I will present the method to make one of these scales [Hour] very easily, \& a Table which will supplant the other scale [Latitude].
By introducing this novel method, Castillon is able to ameliorate one drawback of dialing scales: viz. any given set of scales will not work for dials of all sizes. Using Castillon's technique, only the hour scale is needed and it is drawn to exactly the appropriate length to fit the dialist's intended design.


Begin by drawing north-south (NO) and eastwest (OE) lines meeting at $O$, the intended center of the dial. See Fig. 1 where we illustrate only the afternoon quadrant. Let N be any convenient point on NO and draw line NE from N to a point on OE such that the angle ONE (i.e. $\psi$ ) equals the value for your latitude shown in a table provided by Castillon. The values of the table are easily calculated as: $\psi=\tan ^{-1}(\sin \varphi)$.

Now draw lines NC and EC, both at $45^{\circ}$ angles from line NE and directed away from the origin O. From C, the intersection of these lines, draw lines at successive $15^{\circ}$ intervals from NC and EC. Note where each of these lines cuts NE and connect the origin O to the points of intersection. The resulting lines are the correct hourlines for a horizontal sundial.

Fig. 1

By adopting this technique, the dialist easily constructs an hour scale (NE) to whatever size is appropriate for his or her proposed dial, and properly positions it by looking up a single value in a handy table. In the talk I presented in 2009 on Castillon Tables (see digital bonus), I developed a couple alternative


Fig. 2 constructions which use modified versions of Castillon's idea. However, rather than reproducing those here, I will simply show a method to avoid the use of any table whatsoever - thus turning this construction into one which really requires nothing special from the dialist's toolkit, other than a drawing compass, protractor and straight edge.

Begin again with perpendicular lines ON and OE. Having selected an arbitrary point N , draw the arc OD through an angle equal to the intended latitude of the dial, as shown in Fig. 2. From point D, drop a perpendicular line DE to OE.
Draw line NE and this will serve as the properly positioned hour scale, since angle ONE will indeed equal the angle that a consultation with Castillon's Table would have produced.

To see that this is the case:

$$
\tan \psi=\frac{O E}{O N}=\frac{O E}{D N}=\sin \varphi
$$

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## Digital Bonus....

The digital edition of this issue of The Compendium comes with 5 bonus PowerPoint files:
Sundials and Smartphones is the file that Roger Bailey used for his presentation of this issue's article at the Portland conference.

Brasil's Newest Sundial is the presentation by James Stegenga, showing a dial he recently completed in Marataizes, Brasil.

Time To Reflect is Ken Clark's latest offering giving details of the cross dials he has been designing.
Digital Gnomon Update is Bob Kellogg's update on his project to make a much improved digital gnomon; he originally reported on this via video at the Victoria conference and subsequently in an article: Designing A Digital Sundial, The Compendium, 23(1):32-39, Mar 2016.

Equants Scales \& Castillon Tables is from Fred Sawyer's talk at the 2009 NASS conference in Portland OR. It forms the basis of Fred's two articles in this issue of The Compendium.

## Sundials and Smartphones <br> Roger Bailey (Sidney BC)

When preparing for an extended trip abroad I realized that schlepping all the "necessary" electronic paraphernalia has become a nuisance. Typically this included a digital camera, a GPS, cell phone, a computer, net book or tablet, and perhaps an e-book reader and MP3 player. Add to this load all the chargers, adaptors, earphones and connecting wires. Add again the weight of books and maps. We prefer to travel with carry-on luggage only. With all the electronic gear, it became impossible to meet the size and weight limitations for carry-on luggage. Observing that most people were now travelling with smartphones, I wondered if a modern compact smartphone could meet all our travel needs. These included staying in contact with family and friends by email and phone, navigating with maps, guides and GPS when driving and hiking, documenting where we had been and what we saw with pictures, notes and locations, booking accommodations, flights and car rentals, checking news and weather. With smartphones I discovered that there is an App for almost everything, but at a cost. I am very wary of the typical connection fees for full phone and internet data, perhaps hundreds of dollars a month. These interests and concerns focused my selection on the following criteria:
Needs:

1. Fully capable smartphone, with camera, GPS, WiFi
2. Android Operating System by Google
3. International: Canada, USA, France, Spain, UK, Portugal etc.
4. Minimum connection costs
a. Prepaid Phone: no long term contract
b. Unlocked: Change SIM cards for different countries
c. No Data Plan
d. WiFi only for internet, email, maps, updates

Wants:

1. Large clear screen
2. Good camera
3. Good GPS
4. Pocket sized
5. Storage for Apps, $8 \mathrm{~GB}, \mathrm{SD}$ card for Maps, Pictures etc.
6. Good battery: multi-day for phone, all day for GPS track logs
7. Low cost $\sim \$ 200$
8. Good Reviews and Ratings

There are many suitable smartphones that meet these needs and wants. My choice was a midrange Motorola, the Moto G 2nd generation. This unlocked phone is 7 cm wide, 14 cm high with a 13 cm HD screen, 1.2 GHz processor, Android 5.0.2 OS, 8 MP and 5 MP cameras, GPSA, tilt and direction sensors.

SIM cards can be changed for prepaid phone services in different countries. SIM cards are typically free or as much as $€ 10$. Minimum prepaid connect time is typically $\$ 10$ to $€ 25$.


Figure 1. Motorola Moto G phone

The power is in the software, the operating system connecting all the hardware: phone, WiFi, cameras, GPS, media players, compass, tilt etc. and a huge variety of low cost Apps specific to your interests.

The Google Apps with the Android OS, as shown in Fig. 2, include G Mail, G Maps, Drive cloud storage, Chrome browser, G Docs, etc. My favorite apps are shown in Fig. 3, including sundials, maps, astronomy and travel. Searching for sundials, I have use the sundial atlas, NASS registry and Google Maps.

Sundial Atlas: Put in any location or let the GPS find your location. A map or satellite view will come up showing the locations of the local sundials registered in Fabio Savian's Sundial Atlas "sundialatlas.eu". The app is by Gian Casalegno. Zoom in to separate individual sundials. Touch on any placemark to bring up details and pictures. I used this app extensively on the Canary Island of Tenerife as the database was excellent. The screenshots in Figs 4 and 5 demonstrate how this app helped to locate the declining reclining dials on the Solar Lab tetrahedron at the Observatorio del Teide on Tenerife.


Fig. 2. Standard Google apps on an Android phone


Fig. 3. A selection of handy sundial and travel apps

Google Maps: Specific "My Maps" overlays for Google Maps are of great value when searching for sundials. I set up a number of such overlays suitable for desktop use but they are even more useful with the Google Maps app for smartphones. As you can see in the screenshots in Fig. 6, I've added the placemarks and links to details and pictures.

The Sundials in Catalonia link https://goo.gl/maps/PLAVA1vqbtK2 opens with the screenshot shown in Fig. 6 center. Zoom in to see details as in Fig. 6 right. Red are the target sundials, yellow suns are the dials we visited, and blue are our home base hotels. SCG, the Societat Catalana de Gnomonica, is an excellent database for locations, descriptions and pictures of 3748 sundials. See www.gnomonica.cat.

Before smartphones, I visited France many times searching for all the existing sundials created by G F Zarbula, from 1830 to 1870 . One of the ways to report on this research is the Google Map of Zarbula's Sundials at https://goo.gl/maps/yxMcPiJQYjE2. Fig. 7 left shows the overall map of the Zarbula zone. You
can zoom in on any region. Touch any placemark to bring up the details, pictures, description and related web links.

Offline Maps: The difficulty with the Sundial Atlas and Google Maps is that the data is not stored in the phone. You need online internet access. To reduce connection costs, I only allow WiFi access. One way to work around this connectivity problem is to start on WiFi , fetch the required map image, and then leave the

Canary Islands Tenerife


Observatorio de Tiede


Solar Lab


Figure 4. Zooming in on a location of interest; Sundial Atlas, app version.

Sundials on Solar Lab


Picture of Sundial


Data Sheet


Figure 5. Location detail and information about a sundial of interest.
app running as you move out of WiFi range. The phone's inbuilt GPS will still track your location over the range displayed.
Some map apps, including Open Streets Maps (osmand.net), offer detailed maps that you can download onto your phone and use offline. I now have maps for BC, Alberta, Washington, Oregon, California, France and Spain. On the map you can enter the location data for your specific point of interest and use the app to navigate to that location. Besides searching for sundials, this app is quite useful for navigation as a car GPS (SatNav), and for finding accommodation.


Figure 6. Examples of map overlays in use with Google Maps

To enter a location, flick to the right or touch the three bar Options icon. Select Search and enter the coordinates of the location; then touch the Favorites star. Enter the Name, Category and Description and touch the down arrow to enter it into My Places. Fig. 8 shows example lists of hotels and sundials entered in Open Streets Maps as Favorites in My Places. Touch the name on the list to open the specific detailed map. Touch the blue direction circle to show a route from your GPS location to your destination. Your route can be saved as a track log.
Sundial Websites: Some websites have been set up to display well on mobile phones using the phone's browser rather than an app. The NASS website www.sundials.org is an excellent example. Search "sundials.org" to bring up the home page with the latest news.
Touch the black square Options icon (Fig. 9 left) to open a menu for other pages on the NASS website including the NASS Registry. Select a state or province and then select the sundial of interest. This will show the description and location of the dial, including the latitude and longitude. Scroll down for more info and pictures (Fig. 10). Touch the picture thumbprint to bring up a large picture. Thanks to Bob Kellogg, the NASS webmaster, for this mobile-friendly website.

Zarbula's Sundials


Zarbula Sundials
Giovanni Francisco Zarbula painted over 100 remarkable sundials in the French Alps near Briancon. Over 50 still survive in. Created: January 28, 2010

VIEW MAP LEGEND

Placemark Details


■ Zarbula Sundials

VIEW MAP LEGEND

Untitled layer
Zarbula's signature sundial, on the home of the Marquise de Bardonnèche. Well restored in 1990 by Tournesol. SAF 0517501

Waymark for more information (http:ill
www.waymarking,com/waymarks/ WM7PR8_Zarbula_Sundial_1840_Vallouise_France)

Picture


Figure 7. Google Maps overlap for Zarbula's Sundials

My Places - My Favorites


Sundial Placemark


Hotel Placemark


Figure 8. Examples of Open Streets Maps (OsmAnd) screen designs
Analyzing Sundials: Once you have found a sundial of interest, the smartphone is very useful to record your find. Take pictures with your phone. If the GPS is on, the location (latitude, longitude and elevation) will be recorded in the detailed properties of the photo. Use the compass app to check the orientation of the sundial and declination of the wall.

The compass app also gives the location data. Use the inclinometer app to measure the slope of the gnomon or the dial base for a reclining dial. Within each app you can send yourself an email with the screenshots of picture, location and orientation as well as adding any notes as text. These emails will be sent to you the


Figure 9. The North American Sundial Society's website adapts well to cell phones.


Figure 10. Detail from the NASS Registry
next time you are connected to the internet, in my case by WiFi. Fig 12 shows examples from the reclining declining sundials at the Solar Observatory on Tenerife.

Unfortunately these compass directions don't make any sense, as the angle between the walls is only $35^{\circ}$, not $60^{\circ}$. Even after calibration, getting stable magnetic readings is difficult. There is a learning curve for every app, including setting preferences.


Fig. 11 A Reclining Declining Sundial on Tenerife
Sundial Apps: The previous discussion focused on searching for sundials. There are several apps that are very useful to analyze and design sundials.

Sol Et Umbra (Sun and Shadow) by Gian Casalegno in Italy is an excellent example of a specific sundial app. An early version is described in the NASS Compendium, 20(2) June 2013. This app starts by determining the date, time and your location. It then shows the same sort of data as our old DOS favorite,


Fig. 12 Cell Phone Compass and Inclinometer Apps


Figure 13. Sol Et Umbra
the Dialist's Companion by Fred Sawyer and Bob Terwilliger back in 1996. Fig. 13 left shows the location and local time including the longitude correction and the Equation of Time. Next is the Sun Ephemeris. Fig. 13 right shows the time in other time systems, Muslim prayer times and the Qibla direction.

Press the three dots (Options icon) to bring up choices including Sundial, Lighting, Sun Path, Position, Time and Date, Preferences and Help.


Figure 14. Representation of a vertical sundial
Sundial designs a sundial based on the location and orientation of the phone. Hold it against a wall to see what a sundial would look like on that wall. This uses the GPS and compass so make sure the GPS is giving a good fix and the compass is calibrated before testing on the wall. Fig. 14 is an example of south facing vertical declining east $3^{\circ}$.

Lighting also uses the location and orientation data to show when the surface will be in sunshine. Fig. 15's vertical dial facing south declining east $\sim 8^{\circ}$ is in sun most of the day except when the sun is behind the dial
in the spring and summer. As the wall declines east there are more sunlit hours in the morning than evening. Note the low position of the sun at 2 pm in mid-January.

Sun Path is the next interesting option in the Sol Et Umbra app. In addition to using the GPS, compass, tilt functions for location and orientation, this calculates the path of the sun and uses the camera to show the path of the sun through the seasons.


Figure 15. Illumination graph from Sol Et Umbra


Figure 16. Sun Path feature of Sol Et Umbra.

This makes it easy to see where the sun rises in the view across the Salish Sea from my home. These screenshots show what a useful app this is for anyone interested in sundials.

Fred Sawyer has brought to my attention two similar sundial apps by Yvon Masse, TpSol and DesCad.

TpSol: Like Sol et Umbra, TpSol simulates a sundial for the location and orientation of the phone. Hold it on a wall to see the lines of a sundial. It also computes the solar time and displays simple ephemeris data like the equation of time, solar declination, solar altitude and azimuth. TpSol is the basic free version as shown in Fig. 17 (left).
DesCad: The premium version ( $\sim \$ 2$ ), DesCad draws the sundial based on the location, orientation and gnomon. Scroll down to see the description and the coordinates for the hour lines displayed. Options include hours, arcs, gnomon, and type of sundial. This app remains a work in progress promising added features like the export of the design as DXF and Calcad files.

All these apps use the phone's magnetic compass. Even with careful calibration, the compass readings are often in significant error. Treat the designs based on compass data as illustrative rather than accurate.
LunaSolCal: Another very useful solar-oriented app is LunaSolCal by Volker Voecking Software. The Overview tab gives the time of sun and moonrise and set for your location and time. The Sun tab gives the time, azimuth, and altitude for sunrise, sunset and noon as well as the Equation of Time, solar declination, length of day and twilight times. Flick to the next panel to see a plot of the sun's path and current position. Flick again for the text panel giving data over a range of dates. The Seasons panel displays the date and time of the seasons, solstices and equinoxes. The Moon tab shows similar data for the moon and its phases. See Fig. 18 for screenshots showing some of these features.

TpSol Home Screen


DesCad Home Screen


DesCad Scroll


Sundial of standard time. Correction to get the legal time: +00 h 11
The sundial is of boreal type, the style stands on the plane of the sundial towards the celestial North Pole.

The center of the sundial, junction point of the hour lines and of the style, is inside the sundial.
Center position: ( $-158.5 ; 40.1$ )
Length of the style: 137.9


Figure 17. Screenshots of TpSol and DesCad, by Yvon Masse.

Sun Data


Sun Path


Moon Data


Figure 18. LunaSolCalc, by Volker Voecking Software.
Conclusions: These are some of the smartphone features and sundial oriented apps that I have found very useful. There are many choices of Android and iOS devices including smartphones, tablets. The range of apps available is amazing. Whatever your interest, there is an app for it.

Apps are also unbelievably cheap. Most are free but may carry advertising banners. I have only paid for a couple of apps, $\$ 0.99$ to download some extra offline maps and $\$ 2.09$ for the DesCal sundial app.

For security and privacy I have installed no financial apps for banks and investments or social apps like Facebook, Twitter or LinkedIn. Email accounts are password protected and there is no password cheat sheet for our various accounts and websites.

The major cost is the initial purchase price of an unlocked smartphone. The connection costs can be minimized by buying prepaid phone time without a contract and avoiding internet connection via the phone. At home I bought a SIM card and low cost prepaid phone time at the local convenience store. Their prices are less than half the costs through one of the big regular phone/TV/internet service providers. A $\$ 25$ top up lasts a year vs. $\$ 100$ minimum with the regular suppliers. Costs in Europe are similar, typically $€ 25$ for a SIM card and prepaid time, good for the month as an emergency phone.

Smartphones offer tremendous utility and value. The apps have facilitated many things that were difficult, time consuming and expensive in the past. I recommend adoption for those willing to accept the initial purchase cost and steep learning curves.

Roger Bailey rtbailey@telus.net

## A Delight Still To Chastened And Wiser Men <br> The Leighton Buzzard Observer (UK)

Tuesday 23rd May 1916: The introduction of Daylight Saving Time for the first time last Sunday has not been met with universal approval. The Leighton Buzzard Observer has published a letter from Mr. C. J. Minter of Aspley Guise, who vehemently opposes the change in the following (considerably abbreviated) terms:
"The sundial is an object of interest in many an old-fashioned garden, combining as it were philosophy with beauty, both of which are 'a joy forever.' With the advent of the Daylight Saving Order, however, the question has been asked, 'What shall we do with the sundials?' We would say, just wait. For people will come back to reason and common sense and to the old sundial once again after the 'Daylight Saving Scheme' is dead and forgotten. Were this measure to continue in force, it would be a matter for very deep regret, because it must inevitably work great hardship in a vast number of cases.
People have not taken this measure seriously. It has seemed too utterly ridiculous to merit their attention, and has been regarded more as a half ludicrous joke. You have to say, as it were, to an hour at one time, 'Quick-change, and be gone!' and at another, 'Halt - backwards!' - and make the hour repeat itself. Was there ever a more clumsy, a more childish, a more fatuous device conceived? Surely, it is the most quixotic measure ever brought before our legislators. And all to what purpose? To do something which might have been done (and is being done by many, for numbers of firms have arranged with their employees, to go to work at a reasonably earlier time in the summer than in the winter months, without altering the clocks) in a sound and sensible manner, and not by a method which shows such a scant semblance to reason as to cause one to doubt if there be even a vestige of such a commodity about it.

People will not even know when they were born, nor which is one day and which another, with the vanishing and reappearing of hours at the waving of the clock magician's wand, instead of relying, as we have done hitherto, on the accuracy of the stars. ... There is a possibility of even those who are now so strongly in favour of this Daylight Saving Scheme, presently growing so weary of it as to devise a scheme whereby our legislators shall move the hands of the clock in the opposite direction and return again to correct time. Meanwhile, the sundials should on no account be interfered with, for they will again come to their own, and be a delight still to chastened and wiser men."

Source: http://bedshomefront.blogspot.com/2016/05/what-shall-we-do-with-sundials.html

The Tove's Nest....


In Memoriam - Jim Morrison (1941-2016)
Jim Morrison passed away on April 8, 2016 at his home in Rehoboth Beach, DE. Jim was born in 1941 in Oklahoma. He graduated from Oklahoma State University with a degree in Engineering in 1963. After graduation, Jim moved to the Los Angeles area to work for North American Aviation on engineering projects for the upcoming Apollo program. After moving to Fort Worth to begin his long IBM career, he earned his Master of Applied Science degree from Southern Methodist University and started his family. After retiring from IBM, Jim continued his interest in the history of astronomy. His early interest in sundials led to a deep passion for the astrolabe. He made working astrolabes for countless students and others, and his book about their design and construction is highly praised and very popular. He also participated in researching a 1769 scientific expedition during that year's transit of Venus that led to erection of a historical marker in Lewes, DE.

Counting Sundials at Bowdoin Fred Sawyer fwsawyer@aya.yale.edu


One of the highlights of the sundial tour at the Portland ME conference in June was our visit to the three declining dials (east - shown at left, south - on our cover, and west - shown below) on Hubbard Hall at Bowdoin College, in Brunswick Maine. The east dial indicates time from sunrise to noon each day. As we examined it, Bob Kellogg remarked that it probably was not a familiar site to the students since they seldom were awake at those early morning hours. Sure enough, later that day we were given a copy of an article in the Bowdoin Daily Sun newspaper (http://goo.gl/HtT0Tc) announcing our visit: These aficionados are scheduled to arrive Friday to cast their gaze upward toward vertical sundials adorning the sides of Hubbard Hall-one on the south end; the other on the west side. Evidently no one ever notices the east dial!

Youngest NASS Members Fred Sawyer fwsawyer@aya.yale.edu
Shortly after viewing the great coverage (http://sundials.org/index.php/features/nass-conferences) we got on Portland's WCSH Channel 6 for the NASS conference, my two grandchildren Clara and Adam suddenly realized that NASS is real and not just a figment of Grampa's vivid imagination! They immediately decided that they should become NASS' youngest members and proclaim to everyone that they love sundials.
Of course, I was pleased with this development, but I stressed to them that they would first have to build and test their own sundials before they could be considered members! The result was a marathon session in which they put together their own custom-designed equatorial dials. (See our back cover.)





[^0]:    * Compendium... "giving the sense and substance of the topic witbin small compass." In dialing, a compendium is a single instrument incorporating a variety of dial types and ancillary tools.

[^1]:    ${ }^{1}$ http://www.math.ubc.ca/~cass/courses/m446-03/pl322/pl322.html

[^2]:    ${ }^{1}$ Marcus Vitruvius Pollo was a Roman architect, ca. 50 B.C. to 14 A.D.

